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An efficient CDMA decoder for correlated information sources

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Abstract. We consider the detection of correlated information sources in the ubiquitous code-division multiple-access (CDMA) scheme. We propose a message-passing based scheme for detecting correlated sources directly, with no need for source coding. The detection is done simultaneously over a block of transmitted binary symbols (word). Simulation results are provided, demonstrating a substantial improvement in bit error rate in comparison with the unmodified detector and the alternative of source compression. The robustness of the error-performance improvement is shown under practical model settings, including wrong estimation of the generating Markov transition matrix and finite-length spreading codes.

Keywords: message-passing algorithms, error correcting codes

Contents

1. Introduction

Code-division multiple-access (CDMA) is a core technology of today's wireless communication employing data transmission between multiple terminals and a single base station. Usually in the uncoded CDMA literature [1], the binary information source, modulated for transmission over the channel, is assumed to be taken from an *unbiased* identically independently distributed (i.i.d.) random process.

In practice, however, a substantial level of redundancy can often be observed in real-life uncoded sources (e.g., uncompressed binary images) which can be viewed as a (global) bias in the generating Bernoulli distribution or as correlations between the binary symbols. These correlations can be considered as a *local* bias on a certain symbol in the generating Bernoulli distribution. In such cases, a binary source encoding is used. A source encoder is said to be optimal if it can eliminate all source redundancies and eventually generate unbiased outputs. However, most existing practical source encoders, which are typically fixed-length encoders, are sub-optimal. Besides the extensive complexity of the encoder, its output still contains a certain level of bias or correlations, which can be further exploited in the transceiver design.

As for coded systems, it has been shown [2] that the empirical distribution of any 'good' error-correcting code converges to the channel's capacity-achieving input distribution. A 'good' code is a code approaching capacity with asymptotically vanishing probability of error. Hence, well-coded information sources, in the common case of a binary input additive white Gaussian noise (BI-AWGN) CDMA channel, must be unbiased, as the capacity-achieving input distribution of the BI-AWGN channel is Bernoulli 1/2 [3]. However, bias (global and local) can be found in practical coded CDMA systems in which codes that are not so 'good' are being employed, e.g., systematic Turbo codes [4]–[6], for which the systematic component of the code entails bias.

An approach different than source compression for handling CDMA with biased sources was recently introduced [7]. In the scheme presented the source bias m was assumed to be estimated by the receiver and used for modifying both the naive single-user matched-filter (SUMF) and state-of-the-art CDMA multiuser detectors (MUD). That modification outperformed the alternative of applying source coding by detecting modulated biased sources directly, with no need of source encoding.

J. Stat. Mech. (2009) P07039

A substantial level of redundancy can also be viewed as correlations between the binary symbols with no (global) bias. In this case the aforementioned scheme is useless. Still the correlations, which contain local bias, can be used for an improved detection.

Therefore, tuning CDMA detection for correlated sources is of major importance in both coded and uncoded settings. In this paper, we examine the commonly used random spreading scheme, which lends itself to analysis and describes CDMA with long signature sequences well. We suggest a scheme for handling CDMA with correlated sources, rather than using source compression.

We propose a scheme in which the source correlations are assumed to be estimated by the receiver, or reported periodically to the receiver via an auxiliary low-rate channel. The source correlations are used for evaluating the applied local bias, m, on each transmitted binary symbol, and this is followed by adding a correction factor of the form $\tanh^{-1} m$ to the multiuser CDMA detector [7], recently introduced by Kabashima [8]. The proposed scheme outperforms the alternative of applying source coding throughout a wide range of practical correlation values. Also the scheme derived is shown to yield an improved error performance not only for a large system limit (the limit where the number of users and spreading codes tend to infinity but with a fixed ratio, which is defined as the system load β), but also for finite-length spreading codes. Note that the proposed scheme is not optimal, yet heuristically presents a good error performance and convergence rate. The BER performance may be affected due to the finite-size effect of the transmitted word of each user, where the correlation length is comparable. Also it may be influenced in the case where the applied local bias leads to a non-optimal fixed point of Kabashima's algorithm.

2. CDMA with correlated information sources

Consider a K-user synchronous direct-sequence binary phase shift-keying (DS/BPSK) CDMA system employing random binary spreading codes of N chips over an additive white Gaussian noise (AWGN) channel. The received signal of such a system can be expressed as

$$y_\mu = \frac{1}{\sqrt{N}} \sum_{k=1}^{K} s_{\mu k} b_k + n_\mu, \tag{1}$$

where $s_{\mu k} = \pm 1$ ($\mu = 1, \ldots, N$, $k = 1, \ldots, K$) are the binary spreading chips being independently and equiprobably chosen. The deterministic chip waveform is assumed to be of unit energy; n_μ is an AWGN sample taken from the Gaussian distribution $\mathcal{N}(0, \sigma^2)$; b_k is the (possibly coded) information source binary symbol transmitted by the kth user and is modeled by a Markov process, generated by a Markov transition matrix. In this paper we demonstrate our scheme for information source binary symbols which are modeled by a two-state (2S) Markov process, generated by the following Markov transition matrix:

$$\mathbf{T}_{ab} = \begin{pmatrix} T_{-1-1} & T_{-11} \\ T_{1-1} & T_{11} \end{pmatrix} \tag{2}$$

where T_{ab} $(a, b = \pm 1)$ is the probability of transmitting a binary symbol b following the transmission of a binary symbol a. We assume that each user sends a word; each is assembled from L symbols.

J. Stat. Mech. (2009) P07039

We work under two scenarios regarding the Markov transition matrix. In the first scenario the matrix is assumed to be known to the receiver as side information, for example, after reported via an auxiliary low-rate feedback channel from the receiver. The second scenario refers to the case where there is no side information channel. In this case the estimation of the Markov parameters (T_{ab}) can be done in each iteration of the decoding process according to the tentative value of the decoded words of all the users. According to the probabilities of each decoded binary symbol for being ± 1, the probabilities of decoding a binary symbol b after a (i.e., T_{ab}) can easily be evaluated.

We assume a perfect power-control mechanism yielding unit energy transmissions. Also we assume a situation where N and K are large, yet the system load factor $\beta = K/N$ is kept finite.

The goal of the MUD is to simultaneously detect binary symbols $b_1, b_2, \ldots, b_K$ after receiving the signals $y_1, y_2, \ldots, y_N$. The Bayesian approach offers a useful framework for this. Assuming that the binary signals are independently generated from the unbiased distribution, the posterior distribution from the received signals is provided as

$$P(\mathbf{b}|\mathbf{y}) = \frac{\prod_{\mu=1}^{N} P(y_\mu|\mathbf{b})}{\sum_{\mathbf{b}} \prod_{\mu=1}^{N} P(y_\mu|\mathbf{b})} \tag{3}$$

where

$$P(y_\mu|\mathbf{b}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-1}{2\sigma^2}(y_\mu - \Delta_\mu)^2\right] \tag{4}$$

and $\Delta_\mu = (1/\sqrt{N}) \sum_{k=1}^{K} s_{\mu k} b_k$.

Recently, Kabashima [8] has introduced a tractable iterative CDMA MUD which is based on the celebrated belief propagation algorithm (BP [9, 10]). This novel algorithm exhibits considerably faster convergence than conventional multistage detection [11] without increasing computational cost significantly. It is considered to provide a nearly optimal detection when the spreading factor N is large and the noise level is known. Like for multistage detection, at each iteration cycle t this detector computes tentative soft decisions η_k^t for each user transmission, of the form

$$\eta_k^t = \tanh(h_k^t). \tag{5}$$

The parameters η_k^t and h_k^t are coupled and being iteratively computed using the following recipe:

$$U_k^t = A^t \sum_{l=1}^{K} W_{kl} \eta_l^t + A^t \beta (1 - Q^t) U_k^{t-1}, \tag{6}$$

$$h_k^{t+1} = R^t h_k^0 - U_k^t + A^t \eta_k^t (1 - Q^t) U_\mu^{t-1}, \tag{7}$$

$$R^t = A^t + A^t \beta (1 - Q^t) R^{t-1}, \tag{8}$$

where $W_{kl} \triangleq \sum_{\mu=1}^{N} s_{\mu k} s_{\mu l}/N$, $Q^t \triangleq \sum_{k=1}^{K} (\eta_k^t)^2/K$, $A^t \triangleq (\sigma^2 + \beta(1 - Q^t))^{-1}$ and tentative hard decisions are taken by $\hat{b}_k^t = \mathrm{sgn}(\eta_k^t)$. Producing $\hat{b}_k^t \equiv \hat{b}_k^{t+1}, \forall k$, serves as the convergence criterion.

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J. Stat. Mech. (2009) P07039

3. Improving detection for correlated sources

In order to exploit all the possible knowledge for the detection of each symbol, let all the words of all the users $(b_k^l, b_k^{l+1}, \ldots, b_{k+1}^l, b_{k+1}^{l+1}, \ldots,$ where $k = 1 \cdots K$ and $l = 1 \cdots L)$ be transmitted altogether to the receiver (total amount of $L \times K$ binary symbols). As a result, in the decoding process, the decoder has a maximum knowledge of the correlations of b_k^l (in our case with b_k^{l-1} and b_k^{l+1}). As described in (1) the transmitted binary symbols are modulated over the AWGN channel. In this case, of transmitting all the words of all the users, we reformulate (1) to have the new form of the received signal

$$y_\mu^l = \frac{1}{\sqrt{N}} \sum_{k=1}^{K} s_{\mu k} b_k^l + n_\mu. \tag{9}$$

With the received signals, the decoder applies a SUMF according to

$$h_k^l = \frac{1}{\sqrt{N}} \sum_{\mu=1}^{N} y_\mu^l s_{\mu k}. \tag{10}$$

Using the output of the SUMF, h_k^l, the decoder evaluates the probabilities of each symbol to be $\hat{b}_k^l = 1$ or $\hat{b}_k^l = -1$ according to

$$q_k^l(\hat{b}_k^l = 1) = \frac{1 + \tanh(h_k^l)}{2} \tag{11}$$

and

$$q_k^l(\hat{b}_k^l = -1) = \frac{1 - \tanh(h_k^l)}{2}. \tag{12}$$

Following equations (11) and (12) and using the Markov transition matrix, T_{ab}, the detector evaluates the applied local bias on each bit using the following recipe:

$$p_k^l(\hat{b}_k^l = 1) = \sum_{a,b=\pm 1} q_k^{l-1}(a) \cdot T_{a1} \cdot q_k^{l+1}(b) \cdot T_{1b}, \tag{13}$$

$$p_k^l(\hat{b}_k^l = -1) = \sum_{a,b=\pm 1} q_k^{l-1}(a) \cdot T_{a-1} \cdot q_k^{l+1}(b) \cdot T_{-1b}, \tag{14}$$

and

$$m_k^l = 2\frac{p_k^l(\hat{b}_k^l = 1)}{p_k^l(\hat{b}_k^l = 1) + p_k^l(\hat{b}_k^l = -1)} - 1. \tag{15}$$

We can now incorporate a correction factor for error-performance improvement, based on the local bias (15), into the SUMF output (10) and finally have a binary decision

$$\hat{b}_k = \mathrm{sgn}(h_k^l + \xi_k^l), \tag{16}$$

$\mathrm{sgn}(\cdot)$ is the hard-decision signum function and $\xi_k^l = (\beta + \sigma^2) \tanh^{-1} m_k^l$ [7].

We can also employ the tractable iterative CDMA MUD ((5)–(8)). In order to adapt the MUD for correlated sources and to the case of detecting all the words of all the users simultaneously, we reformulate (5) to have the new form

$$\eta_{kt}^{l} = \tanh(h_{kt}^{l} + \xi_{kt}^{l}), \tag{17}$$

where $\xi_{kt}^{l} = \tanh^{-1} m_{kt}^{l}$ is a correction factor for error-performance improvement being incorporated within the detection algorithm [7].

After each iteration of the MUD, an evaluation of the probabilities of each symbol for being $\hat{b}_k^l = 1$ or $\hat{b}_k^l = -1$ is assessed according to (11) and (12), followed by a calculation of the local bias which is applied on each symbol (13)–(15) and finally by another iteration of the MUD with the correction factor (17).

4. Results and discussion

In this section, simulation results of the proposed scheme for correlated sources are presented. Unless stated otherwise, all the results are obtained for load $\beta = 0.8$ and $\sigma = 0.8$ while simulation results are averaged over a sufficiently large ensemble of 2000 computer-simulated randomly spread AWGN CDMA samples with long spreading factor $N = 1000$.

Figure 1(a) displays the bit error rate (BER) of the proposed scheme ((11)–(15) and (17)) as a function of the relative location of a certain binary symbol in the detected word. The relative location of a certain bit is its location compared to the beginning of the detected word, divided by the length of the detected word. For example, in the case where the length of the detected word is $L = 30$, the relative location of the third bit is 0.1. The results were obtained for a Markov transition matrix with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 0.8$ and for several lengths of words $L = 10, 20, 30, 40, 60, 80, 100, 120$. Results indicate that the portion of the word that has a substantial and stable improvement of the BER increases with the size of the word. This tendency can be explained by using the correlation length of the Markov process, $\xi = 1/\ln(\lambda_1/\lambda_2)$. The effect of a wrong detected bit over other bits in the word vanishes for a length greater than ξ. For example, the correlation length of the aforementioned generating Markov transition matrix is $\xi = 4.48$. Thus, for a detected word with length of $L = 120$ more than 95% of the bits have a (considerably) low and stable BER, while on the other hand for a detected word with length of $L = 10$, we can hardly see a saturation of the BER. Similar results were also obtained for small ($N = 25$) CDMA systems and for other Markov transition matrices.

An interesting feature of the results in figure 1(a) is the convergence to saturation of the BER as a function of the length of the detected word. Figure 1(a) shows that the convergence to the asymptotic BER decreases with the size of the word.

We measured the behavior of the convergence to saturation of the BER as a function of the length of the word, L. Figure 1(b) displays the behavior of the convergence to saturation for correlated sources, generated by several Markov transition matrices. We calculated the convergence to saturation as the relative position in the detected word where the BER (which is displayed in figure 1(a)) is 20% above the level of the minimum BER.

J. Stat. Mech. (2009) P07039

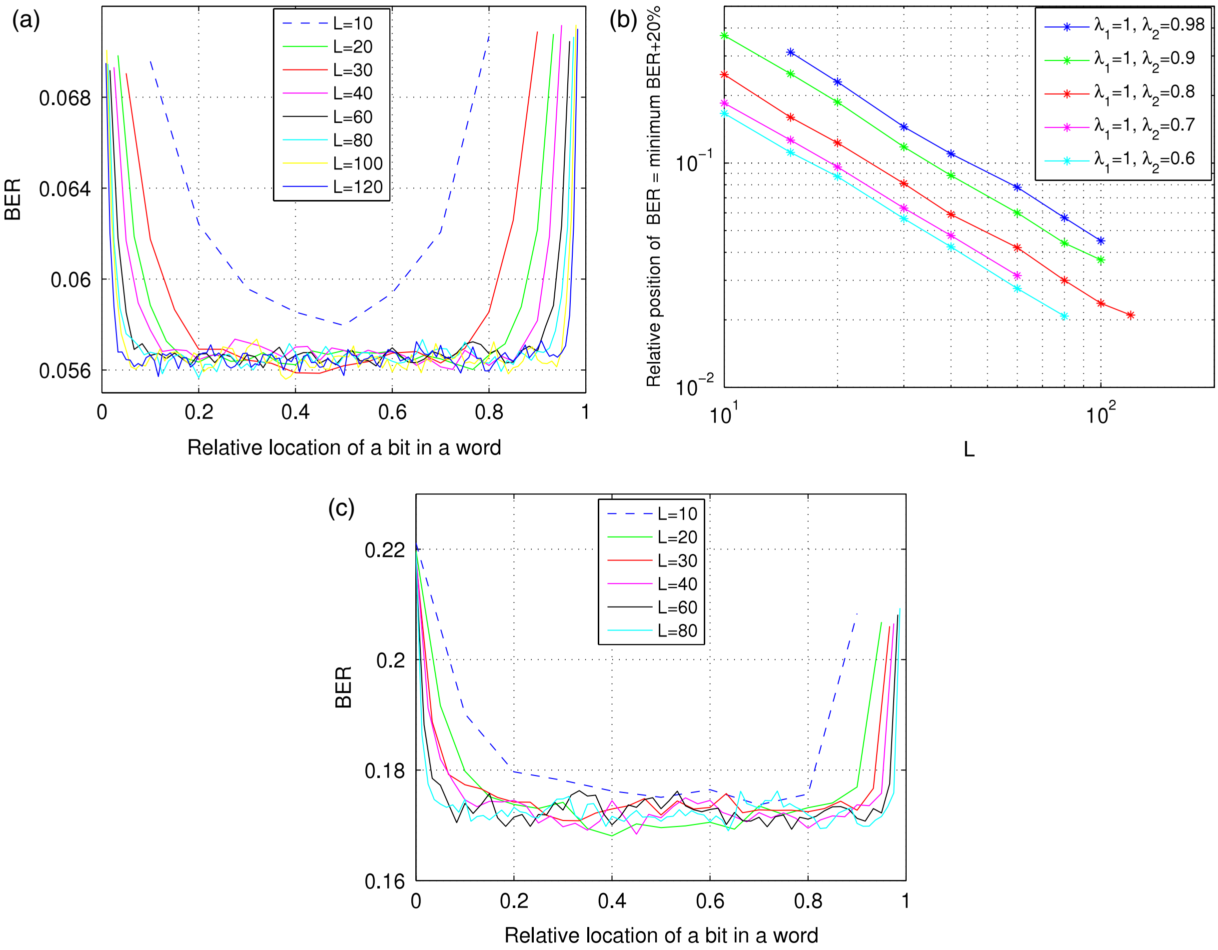

Figure 1. (a) Bit error rate (BER) versus the relative location of a bit in a word for $N = 1000$, $\beta = 0.8$, $\sigma = 0.8$, Markov transition matrix with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 0.8$ and for several lengths of words $L = 10, 20, 30, 40, 60, 80, 100, 120$. Simulation results are averaged over 2000 samples. (b) The relative position in the detected word where the BER is 20% above the minimum BER versus the length of the detected word, L, for $N = 1000$, $\beta = 0.8$, $\sigma = 0.8$ and for several Markov transition matrices. Simulation results are averaged over 2000 samples. (c) SUMF BER versus the relative location of a bit in a word for $N = 1000$, $\beta = 0.8$, $\sigma = 0.8$, Markov transition matrix with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 0.8$ and for several lengths of words $L = 10, 20, 30, 40, 60, 80$. Simulation results are averaged over 2000 samples.

Figure 1(b) indicates that for all correlated sources, generated by different Markov transition matrices, the convergence to saturation of the BER behaves like L^{-1}. Similar results were obtained for CDMA systems with small spreading, $N = 25$, and with other Markov transition matrices.

Figure 1(c) displays the BER of the proposed scheme while using (only) the popular and naive SUMF ((11)–(16)) as a function of the relative location of a certain binary symbol in the detected word. The same behavior as was demonstrated in figures 1(a) and (b) is also presented here.

J. Stat. Mech. (2009) P07039

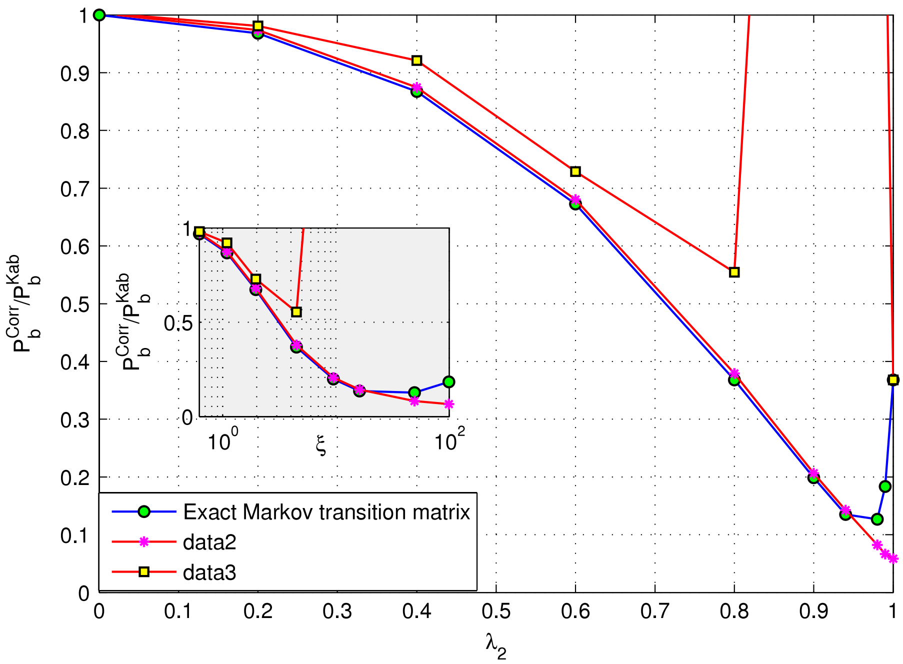

Figure 2. Normalized BER, $P_b^{\mathrm{Corr}}/P_b^{\mathrm{Kab}}$, as a function of the eigenvalues of the Markov transition matrices ($\lambda_1 = 1$ and λ_2) for $N = 1000$, $\beta = 0.8$, $\sigma = 0.8$ and $L = 100$, averaged over 2000 samples. Also drawn are the results of the BER under a mismatch of $\pm 10\%$ of element T_{-1-1}. Normalized BER as a function of the correlation length ξ is drawn in the inset.

Figure 2 displays the normalized BER of the proposed scheme ((11)–(15) and (17)) as a function of the eigenvalues of the Markov transition matrices ($\lambda_1 = 1$ and λ_2) for $L = 100$. The BER is normalized by P_b^{Kab}, the estimated BER if no local bias modification is applied in (17) (i.e., the ordinary MUD of Kabashima is used (5)). A substantial improvement in the BER performance due to the proposed scheme is presented. Notice that for very high eigenvalues ($\lambda_1 = 1, \lambda_2 > 0.995$) the normalized BER grows, since for these eigenvalues $\xi \geq L$ (e.g., $\xi(\lambda_2 = 0.995) \approx 200 > L = 100$), which leads to strong effects of wrong detected bits over the detected word. Figure 2 also presents the normalized BER under a realistic model of a mismatch in the generating Markov transition matrix estimation. Considering a symmetric Markov transition matrix, a mismatch of $\pm 10\%$ of element T_{-1-1} is examined. That is to say, the detector assumes that the probability of detecting $\hat{b}_k^l = -1, \hat{b}_k^{l-1} = -1$ is higher/lower than the true probability. Clearly, the proposed scheme still suggests a substantial improvement in the BER for these practical settings. The sudden growing of the BER for the positive mismatch when $\lambda_2 > 0.8$ is due to the detrimental effect of infinite correlation length.

As stated in section 1, the mainstream alternative to our approach is source coding, or compression [12]. In order to evaluate the attractiveness of the proposed scheme, we compare its bit error probability, P_b^{Corr}, to the bit error probability, P_b^{Comp}, achieved by the nearly optimal Kabashima multiuser detection of the transmission of unbiased (optimally) compressed source bits. We consider the theoretical optimal compression bound without limiting ourselves to any concrete algorithm. The ratio between these two probabilities is given by

$$\frac{P_b^{\text{Corr}}}{P_b^{\text{Comp}}} = \frac{P_b^{\text{Corr}}(T^{\text{Corr}}, \sigma, \beta)}{P_b^{\text{Comp}}(T^{\text{i.i.d}}, \sigma, \beta^{\text{Comp}} = \beta H_b)} = \frac{H_b \cdot P_b^{\text{Corr}}(T^{\text{Corr}}, \sigma, \beta)}{P_b^{\text{Corr}}(T^{\text{i.i.d}}, \sigma, \beta^{\text{Comp}} = \beta H_b)}, \tag{18}$$

where $H_b = -\sum_{a,b=\pm 1} \mu_a T_{ab}^{\text{Corr}} \log_2(T_{ab}^{\text{Corr}})$ denotes the binary source's entropy, μ_a denotes the stationary distribution of the correlated source and T^{Corr} and $T^{\text{i.i.d.}}$ denote the Markov transition matrix and identically and independently distributed (i.i.d.) matrix respectively. Note that as compression results in H_b times fewer compressed bits, they can be transmitted using an H_b times lower bandwidth per bit. In order to exploit the entire bandwidth, the spreading codes should be expanded. We have assumed that the compressed bits are conveyed under an effectively H_b times lower load β. Also, in computing P_b^{Comp} when we assume an optimal source code, asymptotically speaking, a single error in detecting a compressed bit leads, on average, to $1/H_b$ errors in the uncompressed information.

Figure 3 presents simulation results for the BER ratio (18) as a function of the eigenvalues of the Markov transition matrices ($\lambda_1 = 1, \lambda_2$). Interestingly, applying the proposed scheme is superior to applying the optimal compression alternative for all λ_2 of the Markov transition matrices.

Figure 3 also demonstrates the increasing superiority of the proposed scheme over non-optimal compression. A sub-optimal fixed-length source encoder compressing at 5% above entropy is assumed. Again, this curve is obtained using (18), where the optimal compression rate H_b is now replaced by the sub-optimal rate $(1 + 0.05)H_b$.

Note that the same trend of the ratio $(P_b^{\text{Corr}}/P_b^{\text{Comp}})$ is also valid for $\beta > 1$ (for example, $\beta = 1.5$).

Another possible way to compare the proposed scheme and the alternative of source coding is to keep the parameters β and σ constant in both approaches. While the BER of the proposed scheme is calculated as explained above, the BER performance of the source coding is calculated as follows. Applying an optimal encoder/decoder scheme, we can encode a correlated sequence into the same size as for the uncompressed case using the rate Rate $= H_b$. After using the MUD algorithm of Kabashima for the i.i.d. sequence we end up with bit error rate, P_b^{Kab}, which is now the starting point for the optimal decoder of a binary symmetric channel (BSC) with flip rate $f = P_b^{\text{Kab}}$ (we assume that error bits are uncorrelated). The residue error, P_b^{Comp}, after the separation scheme, the MUD of Kabashima and the optimal decoder is given by the following equation:

$$H_b = \frac{1 - H_2(f)}{1 - H_2\left(P_b^{\text{Comp}}\right)}. \tag{19}$$

Typical results are given in the following table.

σ	β	P_b^{Corr}	P_b^{Comp}	$P_b^{\text{Corr}}/P_b^{\text{Comp}}$
0.8	0.8	0.034	0.042	0.81
1	1	0.067	0.111	0.6

J. Stat. Mech. (2009) P07039

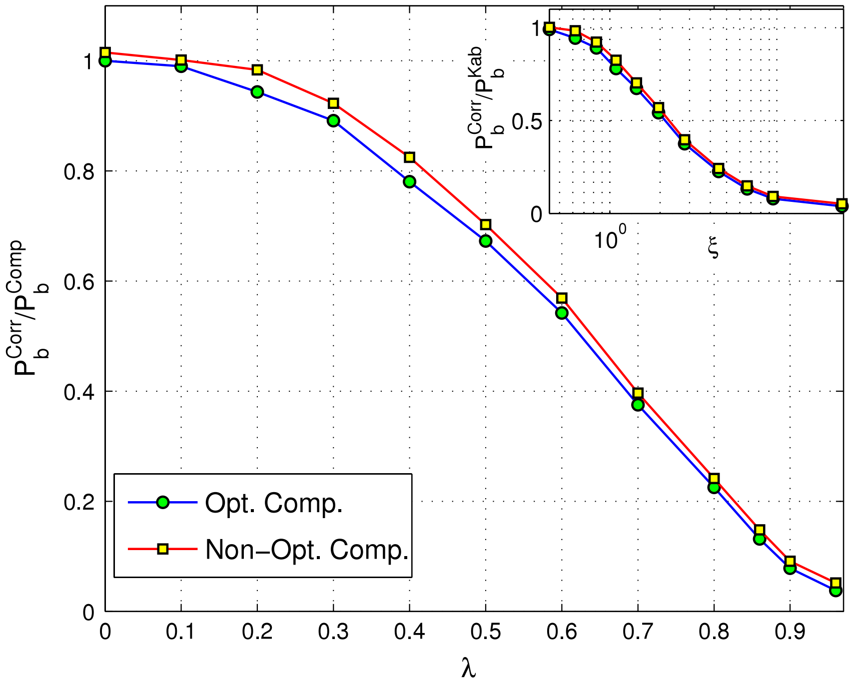

Figure 3. A comparison of the BER obtained in simulations, using the proposed scheme, P_b^{Corr}, versus optimal and non-optimal (i.e., 5% above the entropy) compression, P_b^{Comp}. The ratio ($P_b^{\mathrm{Corr}}/P_b^{\mathrm{Comp}}$) is plotted as a function of the second eigenvalue of the Markov transition matrix, λ_2, for load $\beta = 0.8$, $\sigma = 0.8$ and $L = 30$. The comparison of the BER as a function of the correlation length ξ is drawn in the inset.

Note that the superiority of the proposed scheme relative to the optimal compression alternative is still valid.

5. Summary

We have introduced a new scheme for detecting correlated sources directly, with no need for source coding, by using a message-passing based multiuser detector. The detection was applied simultaneously over a block of transmitted binary symbols. The BER improvement for a transmitted block when using the suggested method has been demonstrated. We have exhibited simulation results which demonstrated a substantial BER improvement in comparison with the unmodified detector case and the alternative case of source compression. The robustness of our scheme has been shown, under practical model settings, including wrong estimation of the generating Markov transition matrix at the receiver and when using finite-length spreading codes.

An important factor in implementing such a scheme is its computational cost. The complexity of Kabashima's iterative detector [8] when detecting a bit for a single user is $\mathcal{O}(tN)$, where t is the number of iterations required for convergence. Adapting it for the case of K users, where each transmits a word of length L, the complexity is $\mathcal{O}(LtKN)$. In the proposed scheme, where all the words of all the users are decoded simultaneously, the complexity is $\mathcal{O}(L\tilde{t}KN)$. Simulations indicate that t and $\tilde{t}$ are comparable, although applying the existing algorithm [8] will result in a higher BER. The proposed scheme manages to incorporate the knowledge of the correlations between the transmitted binary symbols in the existing algorithm [8] without increasing computational cost. Though the proposed scheme is heuristic, it still obtains a better BER performance relative to the unmodified detector and the alternative of source compression.

Another important factor is the updating scheme of the local bias calculation applied to each binary symbol over the transmitted block of information. Four updating schemes were tested on the proposed detector: the parallel updating scheme (PUS), the sequential updating scheme (SUS), the back–front updating scheme (BFUS) and the random-sequential updating scheme (RSUS). Simulations indicate that the best BER performance and convergence rate, in iterations, are achieved when using SUS or BFUS.

References

[1] Verdú S, 1998 *Multiuser Detection* (Cambridge: Cambridge University Press)
[2] Shamai S and Verdú S, 1997 *IEEE Trans. Inf. Theory* **43** 836
[3] Tanaka T, 2002 *IEEE Trans. Inf. Theory* **48** 2888
[4] Berrou C, Glavieux A and Thitimajshima P, 1993 *IEEE Int. Conf. Commun. (Geneva)* p 1064
[5] Zhu G C and Alajaji F, 2002 *IEEE Commun. Lett.* **6** 64
[6] Zhu G C and Alajaji F, 2001 *39th Allerton Conf. on Commun. Control and Computing (Monticello, IL)*
[7] Efraim H, Yacov N, Shental O, Kanter I and Saad D, 2007 *J. Phys. A: Math. Theor.* **40** 9977
[8] Kabashima Y, 2003 *J. Phys. A: Math. Gen.* **36** 11111
[9] Pearl J, 1988 *Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference* (San Francisco, CA: Morgan Kaufmann Publishers)
[10] Opper M and Saad D, 2001 *Advanced Mean Field Methods: Theory and Practice* (Cambridge, MA: MIT Press)
[11] Andrews J G, 2005 *IEEE Wireless Commun.* **12** 19
[12] Cover T M and Thomas J A, 1991 *Elements of Information Theory* (New York: Wiley)

J. Stat. Mech. (2009) P07039