Synchronization of Mutually Coupled Chaotic Lasers in the Presence of a Shutter

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(Received 3 October 2006; published 13 April 2007)

Two mutually coupled chaotic diode lasers exhibit stable isochronal synchronization in the presence of self-feedback. When the mutual communication between the lasers is discontinued by a shutter and the two uncoupled lasers are subject to self-feedback only, the desynchronization time is found to scale as $A_d\tau$, where $A_d > 1$ and τ corresponds to the optical distance between the lasers. Prior to synchronization, when the two lasers are uncorrelated and the shutter between them is opened, the synchronization time is found to be much shorter, though still proportional to τ. As a consequence of these results, the synchronization is not significantly altered if the shutter is opened or closed faster than the desynchronization time. Experiments in which the coupling between two chaotic-synchronized diode lasers is modulated with an electro-optic shutter are found to be consistent with the results of numerical simulations.

DOI: 10.1103/PhysRevLett.98.154101 PACS numbers: 05.45.Vx, 42.55.Px, 42.65.Sf

Chaotic systems are characterized by an irregular motion which is sensitive to initial conditions and tiny perturbations. Nevertheless, two chaotic systems can synchronize their irregular motion when they are coupled [1]. When the coupling is switched off, any tiny perturbation drives the two trajectories apart. The separation is exponentially fast, and it is described by the largest Lyapunov exponents of a single system.

In this Letter, we show that the trajectory dynamics of coupled chaotic systems, which also poses time-delayed self-feedback [2], is different. In such systems, the time scale for the separation of the trajectories is found to be much longer than the coupling time. On the other hand, when the coupling is switched on, resynchronization occurs on a faster time scale. We investigate this phenomenon numerically and show the first experiments which support our numerical simulations.

Semiconductor (diode) lasers subjected to delayed optical feedback are known to display chaotic oscillations, and two coupled lasers exhibit chaos synchronization. Different coupling setups such as unidirectional or mutual coupling and variations of the strength of the self- and coupling feedback result in different synchronization states: The lasers can synchronize in a leader-laggard or an anticipated mode [3,4], as well as in two different synchronization states: achronal synchronization, in which the lasers assume a fluctuating leading role, or isochronal synchronization, where there is no time delay between the two lasers' chaotic signals [2,5–8].

In this Letter, we focus on a symmetric setup with time delay between lasers τ_c and self-feedback time delay τ_d. In the event of $\tau_c = \tau_d = \tau$ and for a wide range of the mutual coupling strength σ and the strength of the self-feedback κ, the stationary solution is isochronal [2,5,9]. The quantity which measures the degree of synchronization between the two lasers A and B, with time-dependent intensities I_A and I_B, is the time-dependent cross correlation ρ, defined as

$$\rho(\Delta t) = \frac{\sum_i (I_A^i - \langle I_A^i\rangle)(I_B^{i+\Delta t} - \langle I_B^{i+\Delta t}\rangle)}{\sqrt{\sum_i (I_A^i - \langle I_A^i\rangle)^2 \sum_i (I_B^{i+\Delta t} - \langle I_B^{i+\Delta t}\rangle)^2}},$$

where the summation is over times indicated by i. Isochronal synchronization is defined by the cross correlation, ρ, having a dominant peak at $\Delta t = 0$.

We control the mutual coupling between the lasers by a shutter, located at a distance $c\tau/2$ from each one of the lasers, where c is the speed of light. When the shutter is open, the two lasers are mutually coupled with strength σ and with self-feedback κ. When the shutter is closed, the self-feedback κ_e is increased to a value of $\sigma + \kappa$ so that the total feedback in the open or closed states remains a constant. This is required so as to prevent a sudden drop in the overall feedback, which would typically destroy the synchronization immediately [10].

The two quantities of interest in this Letter are the desynchronization time t_d and the resynchronization time t_r. The desynchronization time is defined as the average required time for the correlation to decay to $C_d\rho(0)$, where $C_d < 1$. The time is measured from the moment the shutter is closed, and $\rho(0)$ is the average correlation in the isochronal phase. The resynchronization or recovery time is measured after the shutter has remained closed for a long period, the two chaotic lasers are uncorrelated, and $\kappa_e = \kappa + \sigma$. The shutter is then opened, and the self-coupling strength is reduced to κ. The resynchronization time is defined as the average time required, from the shutter opening, for ρ to increase from zero to $C_r\rho(0)$, where C_r is a constant ≤ 1.

To numerically simulate the system, we use the Lang-Kobayashi rate equations [11] that are known to describe a chaotic diode laser. The dynamics of the lasers are given by coupled differential equations for the optical field E, optical phase Φ, and excited state population n for each laser.

In the calculation, we used values and parameter definitions as those used in Refs. [2,5,10], where the experimentally tunable parameters are κ, σ, and p, which is the ratio of the actual laser injection current to the threshold current.

Figure 1 displays the calculated desynchronization time as a function of τ for $\kappa = \sigma = 50\ \mathrm{ns}^{-1}$, $\rho(0) > 0.99$, $p = 1.2$, and $C_d = 0.8$ and 0.9. Each data point is averaged over 50 samples, and $\rho(0)$ is measured by averaging a sliding window (sliding length is 1 ns) over a length τ, while the solid lines are obtained by a linear fit, $t_d = A_d\tau + \text{const}$. The calculation shows that t_d scales linearly with τ with a slope, which increase as C_d decreases, and is near 7 and 9 for $C_d = 0.9$ and 0.8, respectively. A similar linear scaling of the desynchronization time was obtained for all values of p in the range from 1 to 1.5, where, for a given C_d, t_d decreases with p. The simulation assumes an ideal shutter with instantaneous closing and opening times and also assumes a discontinuous decrease of σ to zero while κ increases to $\kappa + \sigma$. We find that the linear scaling as well as the slope are robust to the following two experimentally necessitated perturbations: (a) a nonideal shutter, which closes gradually over a period of 10–20 nanoseconds, and (b) an imperfectly closed shutter, allowing residual mutual coupling of a few percent of σ while in the closed state.

In the inset in Fig. 1, the average ρ versus time for $\tau = 50$ ns is shown with the shutter closing at $t = 0$. The correlation coefficient clearly does not decay via a typical exponential; rather, it is almost a constant for an initial period (first ~250 ns for the parameters of Fig. 1) and then crosses over to an exponential decay for long times. Because the event of shutter closing or opening takes a time $\tau/2$ to propagate to the lasers, it is reasonable to expect that $t_d > \tau/2$. It is surprising, however, that t_d scales linearly with τ with a prefactor which is significantly greater than 1. It is also not obvious from the simulation [in which $\rho(0) > 0.99$] if such behavior can be observed in an experiment where $\rho(0) \leq 0.9$.

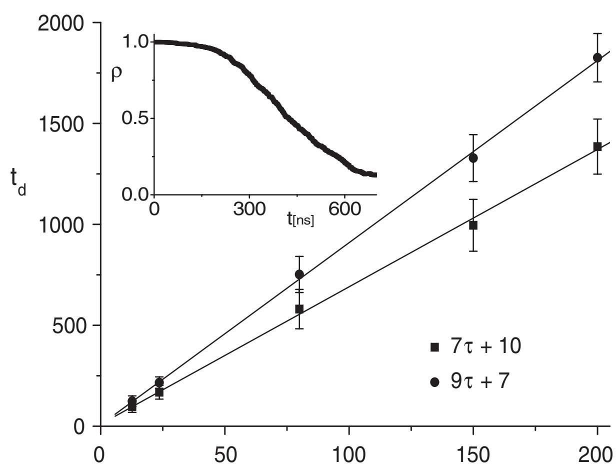

FIG. 1. t_d as a function of τ for $p = 1.2$, $\kappa = \sigma = 50\ \mathrm{ns}^{-1}$, and $C_d = 0.9$ (■) and $C_d = 0.8$ (●). The lines are a result of a linear fit, and the error bars were obtained from 50 independent calculations for each point. Inset: The average ρ as a function of time for the same parameters as in the main figure and $\tau = 50$ ns.

The nearly constant value of ρ after the coupling between the lasers is terminated calls for a theoretical explanation. Let us discuss the synchronization for $\kappa \sim \sigma$, where both lasers are driven by an almost identical delayed signal which is the sum of the self-feedback and the coupling beam. When the mutual coupling is switched off and replaced with stronger self-feedback, the system still feels its synchronized state for a period of length τ, since the system is coupled to its history delayed by a time τ. The only difference caused by the closing of the shutter is that the lasers no longer communicate with each other, and each laser is coupled only to its own state, which contains the memory of the coupled laser history provided by the delayed self-feedback. With time, this memory fades as small differences in the laser output develop. These small differences are amplified as they are fed back to the laser, and this occurs stepwise in time intervals of length τ. For each interval, there is a constant distance between the two trajectories, which increases for the following interval. Only the envelope of these steps is described by the largest Lyapunov exponents but not the dynamics itself. Hence, desynchronization is very slow, with a time scale proportional to τ. On the other hand, when the exchanged beam is switched on again, the electric field of the light in the coupling path between the lasers resynchronizes immediately, yielding an identical feedback signal (for $\kappa = \sigma$), and leads to very fast resynchronization of the lasers, independent of τ. We thus expect the desynchronization time to be insensitive to the values of κ and σ and to scale with τ. In contrast, the resynchronization time should be very sensitive to the difference $\kappa - \sigma$.

The experimental setup, which confirms many of these numerical predictions, is shown schematically in Fig. 2. We use two Fabry-Perot semiconductor lasers emitting at

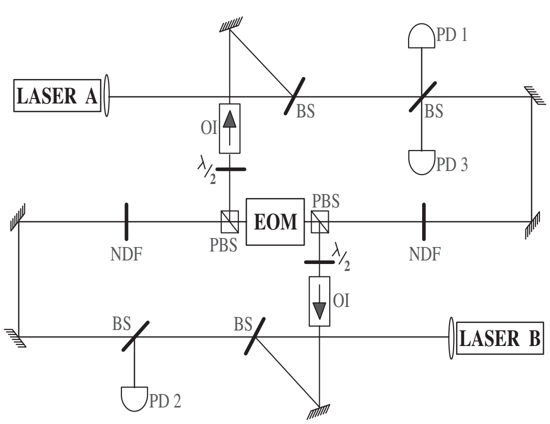

FIG. 2. Schematic of the two mutually coupled lasers. EOM: Electro-optic modulator; BS: beam splitter; PBS: polarizing beam splitter; OI: optical isolator; NDF: neutral density filter; PD: photodetector.

660 nm, selected to have nearly the same threshold current, emission wavelength, and output power and operated close to their threshold currents. The temperature of each laser is stabilized to better than 0.01 K, and the individual laser temperatures are tuned so that the two lasers have nearly identical output wavelengths [9]. The lasers have similar optical feedback and are mutually coupled by injecting a fraction of each one's output power to the other. A fast electro-optic modulator, with a measured closing or opening time of 15 ns, is placed in the middle of the coupling optical path to enable closing and reopening of the mutual coupling. The optical setup is designed to compensate the sudden drop in the overall feedback power when the shutter is closed and the mutual coupling feedback drops to near zero. We thus use the shutter as a polarizing beam splitter which divides the output power of the laser into two parts: one used for the self-feedback and the other for the mutual coupling channels. The opening and closing of the shutter merely changes the ratio of powers in the channels but maintains the overall feedback power at a constant level. This setup, however, does not prevent the change in phase of the laser field when the shutter changes its state. This phase jump decreases the level of synchronization by a small amount (see Fig. 3). The shutter also does not close hermetically, and the leakage power to the mutual coupling channel in the closed state is measured to be $\sim$7% of the shutter open value.

The feedback strength of each laser is adjusted using a $\lambda/2$ wave plate and an optical isolator, which prevents a counterpropagating self-feedback beam from entering the laser diode and also functions as a polarizer. The waveplate-optical isolator (polarizer) combination is set so that the solitary laser threshold current is reduced by about 5% [9]. The lasers produce linearly polarized light with a small unpolarized component which is ignored. The linear polarization of the lasers are aligned to each other by

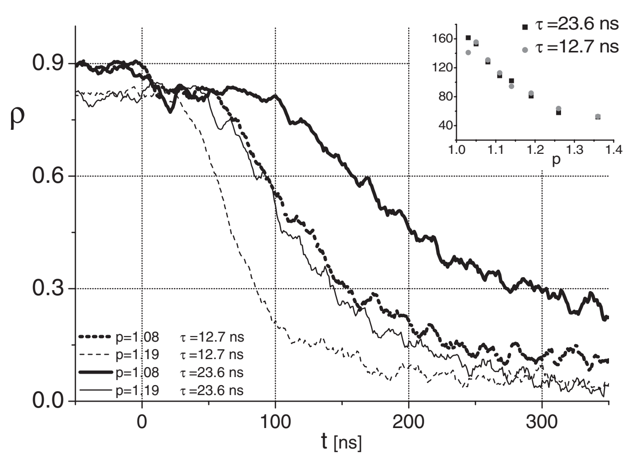

FIG. 3. Experimental results for ρ as function of t for $p = 1.08, 1.19$ and $\tau = 12.7, 23.6$ ns. Each data point was averaged over 500 measurements. Inset: Data collapse, $1.7t_d$ (t_d) measured in nanoseconds versus p for $\tau = 12.7$ ns (23.6 ns).

rotating the diodes in their mounts. The lengths of the self-feedback and coupling optical paths are set to be equal to obtain stable isochronal synchronization [2]. Two sets of measurements are reported here, corresponding to two self-feedback optical paths with $\tau = 12.7$ ns and $\tau = 23.6$ ns, respectively.

Two fast photodetectors (response time <500 ps) are used to monitor the laser intensities, which are simultaneously recorded with a digital oscilloscope (500 MHz, 1 GS/s). The correlation coefficient ρ is calculated by dividing the intensity traces into 10 ns segments (each segment containing 10 points), and ρ is calculated between matching segments and then averaged.

The measured ρ is shown in Fig. 3, where the shutter is closed at $t = 0$. The coupling power decays to its closed level in about 15 ns, limited by the speed of the shutter. The observed decay time is only slightly shorter than the decay time obtained in simulations for the same τ, and, as in the simulations, ρ initially maintains a high and nearly constant value for 50–100 ns, which is much longer than τ. The four data curves shown correspond to different experimental parameters, indicated by the value of p and the two different values of τ.

Figure 3 shows that t_d increases with τ, and the inset, depicting t_d as a function p, indicates that t_d is a decreasing function of p. The inset also demonstrates that the different decay curves all collapse to a single decay curve, independent of p, when scaled by a factor of 1.7, which is close to the ratio of $\tau_1/\tau_2 = 23.6/12.7 \sim 1.86$. Numerical simulations for larger τ also exhibit such data collapse when scaled by τ_1/τ_2, resulting in scaled decay curves which are independent of p. This result and the linear scaling of t_d for a given p indicate $t_d(\tau, p) \propto \tau g(p)$, where $g(p)$ is a function characteristic of the specific diode laser used. For small τ, finite size effects are expected as a result of the positive constant in the linear scaling shown in Fig. 1. Indeed, for $\tau = 12.7$ and 23.6 ns, the numerical results indicate that the average ratio $t_d(\tau_1, p)/t_d(\tau_2, p) \sim 1.75$, which is in surprisingly good agreement with the experimental result of 1.7.

The simulations also indicate that a transition from the low frequency fluctuation regime to the fully developed coherent collapse regime [12] occurs at $p \sim 1.35$, which is close to the experimental value of p (inset in Fig. 3) where t_d becomes almost independent of p. Though the decay time obtained from the simulations is longer than the experimental decay time, this is not surprising, since the simulated systems are initially correlated to a very high level [$\rho(0) > 0.99$], while the initial experimental correlation is $\rho(0) \leq 0.9$.

We now turn to examine the scaling of the resynchronization time t_r as a function of τ. In the simulations, we start with two uncoupled systems with self-feedback $\kappa_e = \kappa + \sigma$. When the shutter is opened at $t = 0$, κ_e is reduced discontinuously to κ, and the mutual coupling is set to σ. For all examined cases, with $\kappa \neq \sigma$, our calculations indicate that t_r also scales linearly with τ. This scaling is

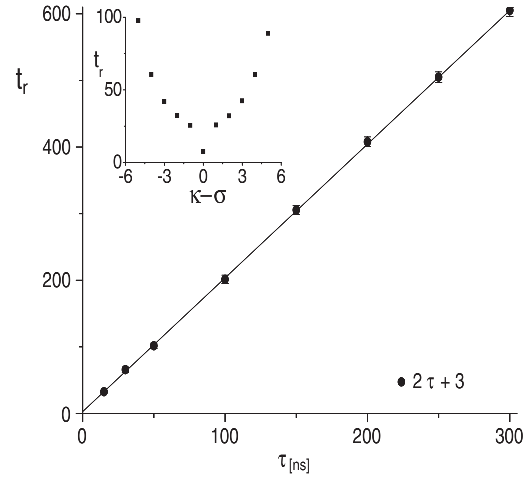

FIG. 4. t_r as a function of τ for $p = 1.2$, $\kappa = 60\ \text{ns}^{-1}$, and $\sigma = 40\ \text{ns}^{-1}$. Inset: t_r as a function of $\kappa - \sigma$ for $\tau = 15$ ns.

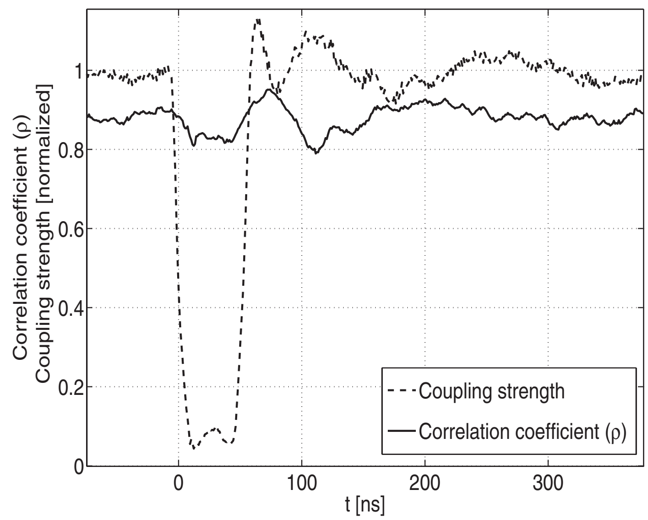

FIG. 5. Normalized coupling strength σ (dashed line) and correlation strength ρ (solid line) as a function of time.

exemplified in Fig. 4 for $C_r = 0.9$, $\kappa = 60\ \text{ns}^{-1}$, and $\sigma = 40\ \text{ns}^{-1}$.

The inset in Fig. 4 displays the resynchronization time as a function of $\kappa - \sigma$ for a given τ. It appears that this difference, rather than the coupling strength itself, is what controls t_r, which scales almost linearly and symmetrically with $\kappa - \sigma$. For $\kappa = \sigma$, t_r is very short, and simulations indicate that it is independent of τ (limited by the fixed size of the sliding window) as expected. In all examined cases, the prefactor of the linear scaling of t_r was found to be $\ll A_d$, indicating $t_r < t_d$. We also observed similar behavior, i.e., $t_r < t_d$, in the experiment, though quantitative determination of the experimental resynchronization time is complicated by ringing in the high voltage electronics used to turn on the modulator and by the fact that the modulator response time is as long as 15 ns, which is comparable to t_r.

Although experimentally we cannot accurately measure the resynchronization time, we show, in Fig. 5, a demonstration of the persistence of the synchronization between the lasers upon repeated closing-opening operations of the shutter. Shown is the typical behavior of ρ while the shutter is closed for $\sim$40 ns and then reopened. The other parameters of the experiment are $\tau = 12.6$ ns and $p \sim 1.08$. The cross correlation coefficient ρ is not affected by the closing or reopening of the shutter and the changing of κ_e and σ, since $t_d > 40$ ns.

We have previously shown [2] that, over a significant operational phase space, two mutually coupled semiconductor lasers can synchronize to each other very well, while an attacker (unidirectional listener) laser cannot synchronize nearly as well with the chaotic signal. In this Letter, we add a significant additional hurdle to the would-be attacker; the communication between the mutually coupled pair can be shut off and the pair nevertheless remain synchronized, while an attacker certainly cannot synchronize while the shutter is closed. Furthermore, the mutually coupled pair can resynchronize, in a short time, before closing off their communication again. Thus, our results for resynchronization or desynchronization times demonstrate the possibility of establishing a reliable chaos-based communication channel even while the communication between the lasers is interrupted by relatively long intervals. We expect that these effects will play an important role in advanced secure communications using mutually chaotic lasers.

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