Synchronization of Mutually Versus Unidirectionally Coupled Chaotic Semiconductor Lasers

Noam Gross[1], Wolfgang Kinzel[2], Ido Kanter[1], Michael Rosenbluh[1], Lev Khaykovich[1]
[1]*Department of Physics, Bar-Ilan University, Ramat-Gan, 52900 Israel, and*
[2]*Institut für Theoretische Physik, Universität Würzbur, Am Hubland 97074 Würzburg, Germany*

Synchronization dynamics of mutually coupled chaotic semiconductor lasers are investigated experimentally and compared to identical synchronization of unidirectionally coupled lasers. Mutual coupling shows high quality synchronization in a broad area of self-feedback and coupling strengths parameters phase-space. It is found to be tolerant to significant parameter mismatch which in case of unidirectional coupling causes lost of synchronization. The advantages of mutual coupling are emphasized in light of its potential use in chaos communications.

PACS numbers:

Chaos communication has drawn much attention in the last two decades. A semiconductor laser subjected to an external feedback displays complex chaotic behavior [1–4]. Two chaotic lasers have shown the ability to synchronize with each other and have proven to be excellent candidates for fast and secure communication[5–7]. Recently, a field experiment using long fiber spans of commercial optical networks has been conducted, in which chaotic optical communications at high transmission and low bit-error rates were reported [6]. In this experiment, a receiver laser was synchronized to a transmitter laser unidirectionally allowing unidirectional information flow only. The advantage of a mutually synchronized system is straight forward: it increases the efficiency of the apparatus by allowing a bilateral conversation. Moreover unidirectional communication is a private-key system where system parameters serve as a secret key. In ref. [8] it was shown that it might be possible to use the synchronization of two mutually coupled symmetric chaotic systems for novel cryptographic key-exchange protocol, whereby secret messages can be transmitted over public channels without using any previous secrets. Recently we made a first step toward realization of such protocol by suggesting a mutually chaos pass filter procedure based on experimental evidence of window of parameters where mutual coupling was shown to be in advantage over its unidirectional counterpart [9]. Here we make an additional step showing robustness of mutually coupled lasers to different parameter's mismatch.

An examination of chaos based synchronization via unidirectional optical coupling points out two types: identical and generalized synchronization [5, 10–12]. The first, also known as anticipated synchronization, appears when the two lasers are subjected to the same optical feedback. One laser simply reproduces the dynamics of the other. The most simple and effective way to achieve this type of synchronization is by setting the transmitter laser's (TL) external feedback strength, κ_t, and the receiver laser's (RL) coupling strength, σ_r, to be equal to each other $\kappa_t = \sigma_r$, while RL external feedback $\kappa_r = 0$ [5]. The second, also known as the isochronous type, requires strong injection (coupling rate) and the RL behaves more like a driven oscillator with it's output responding to the injection signal [13]. This synchronization is robust to channel disturbances [12] and works well even under non identical laser parameters, hence it is favorable for chaos communication [6, 7, 11, 15, 16]. Although the identical synchronization is potentially capable of achieving better synchronization, it is difficult to realize in real systems due to its high sensitivity for parameter mismatch [12–14].

Mutual optical coupling has been explored mostly in a face to face configuration, where the lasers have no external cavities [17–19]. In that scheme, achronal synchronization along with asymmetric physical roles between the lasers was found, even under symmetric operating conditions. Recently, we have introduced mutual optical coupling with self feedback where isochronal synchronization takes place [20], i.e. the lasers are synchronized with zero time delay. A necessary condition for this type of synchronization is that the self feedback round-trip lengths of both lasers will be equal to the coupling length (the distance between them). Considering both lasers are subjected to approximately the same feedback - high quality identical synchronization is achieved.

In this paper we focus on the isochronal synchronization of mutual coupling. We explore the feedback and coupling strength phase-space and show that this identical synchronization has much higher tolerance for parameter mismatch than its unidirectional counterpart. Besides the obvious merits of mutual coupling, such as the bidirectional flow of information, it combines the high correlation offered by the identical synchronization with the robustness typical to the generalized case.

Our experimental setup is depicted in Fig.1(a) and in Fig.1(b) for the mutual and unidirectional coupling, respectively. We use 2 single-mode lasers emitting at 660 nm and operating close to their threshold. Let us emphasis that our lasers are off-the-shelf non-preselected lasers. However, at room temperature they posses a very similar optical wave length ($\Delta\lambda = 0.2$nm) and the same threshold current (43.1mA) and P/I curve (better than 99% match). The temperature of each laser is stabilized to better than 0.01K. The length of the external cavities

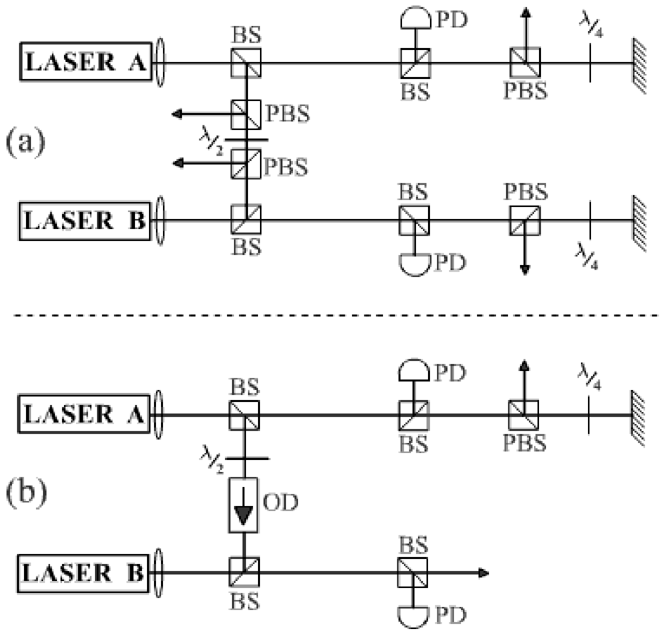

Figure 1: A schematic figure of the two coupled lasers for the mutual (a) and unidirectional (b) cases. BS - Beam Splitter; PBS - Polarizing Beam Splitter; OD - Optical Isolator; PD - Photodetector

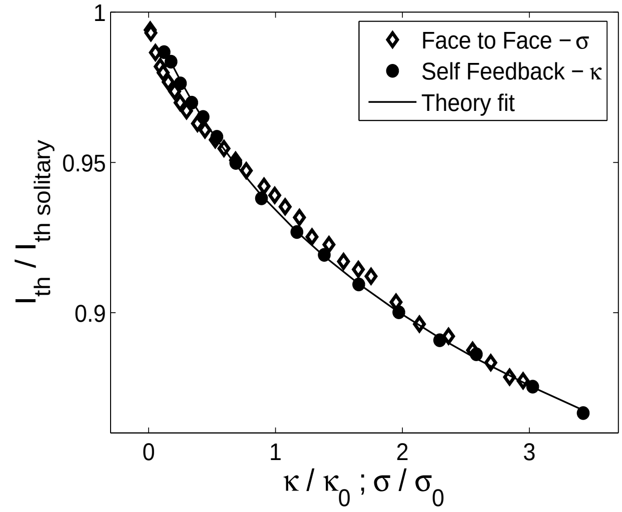

Figure 2: Measurements of the relation between the amount of feedback a laser is subjected to and the effect of reduction in its threshold current. σ_0 and κ_0 are the equivalent for a 6.6% reduction in I_{th}^{sol}. Theoretical fit is according to eq.1 (see text).

is set to 180 cm (round trip time 12 ns). Self feedback strength is adjusted using a $\lambda/4$ wave plate and a polarizing beam splitter. In the mutual coupling experiment (Fig.1(a)), the two lasers (A and B) are mutually coupled by injecting a fraction of each one's output power to the other. Coupling power is adjusted using a $\lambda/2$ wave plate and two polarizing beam splitters. In the second experiment (Fig.1(b)), laser B is coupled unidirectionally to laser A, with unidirectionality ensured by an optical diode (-34 dB). Coupling power is adjusted using a $\lambda/2$ wave plate located in front of the optical isolator. In both cases, coupling optical paths are set to be equal the self feedback round trip path, i.e. there is a 12 ns time delay between the lasers. Two fast photodetectors (response time < 500 ps) are used to monitor laser intensities which are simultaneously recorded with a digital oscilloscope (500GHz, 1GS/s).

We first explore a phase-space of feedback and coupling strength for the mutual coupling synchronization. The feedback power a laser receives is measured based on the fact that a laser subjected to an external feedback exhibits a reduction in its threshold current. The relation between the threshold current, I_{th}, of a laser with an external feedback and the threshold current of the solitary laser, I_{th}^{sol}, is [21]

$$\frac{I_{th}}{I_{th}^{sol}} = 1 - \gamma \ln(1 + \delta \frac{\kappa}{\kappa_0}) \tag{1}$$

where γ and δ are constants, κ is the feedback strength and κ_0 is the feedback strength equivalent to a reduction of 6.6% in I_{th}^{sol}. κ is proportional to the fraction of light power coupled back into the cavity. In Fig.2 (closed circles), this relation is confirmed experimentally. The theoretical fit provides the values $\gamma = 0.0754$ and $\delta = 1.4$ for fitting parameters. Moreover, we found coupling strength between the lasers to be in good agreement with $eq.$(1) as well. To measure coupling strength influence on the laser's threshold current reduction, the lasers were set in a face to face configuration, in which they are exposed to mutual feedback without having an external cavity of their own. Lasers' frequencies were set with the help of temperature control to achieve a maximum overlap between internal modes. The measurement is shown in Fig.2 as open diamonds. Again, σ_0 is equivalent for a 6.6% reduction in I_{th}^{sol} of both lasers, and σ is the coupling strength. We attribute small deviations from the self-feedback case to the residual difference in lasers frequencies. In the following we refer to κ and σ in respect to κ_0 and σ_0.

Fig.3 shows the phase-space of self feedback (κ) and coupling strengths (σ) obtained for mutual coupling. At any point in this phase space, a total feedback amount symmetry is preserved, i.e. $\kappa = \kappa_A = \kappa_B$ and $\sigma = \sigma_A = \sigma_B$. Quality of synchronization between lasers is evaluated with the correlation coefficient [20], ρ defined as follows:

$$\rho = \frac{\sum^i (I_A^i - <I_A^i>) \cdot (I_B^i - <I_B^i>)}{\sqrt{\sum^i (I_A^i - <I_A^i>)^2 \cdot \sum^i (I_B^i - <I_B^i>)^2}} \tag{2}$$

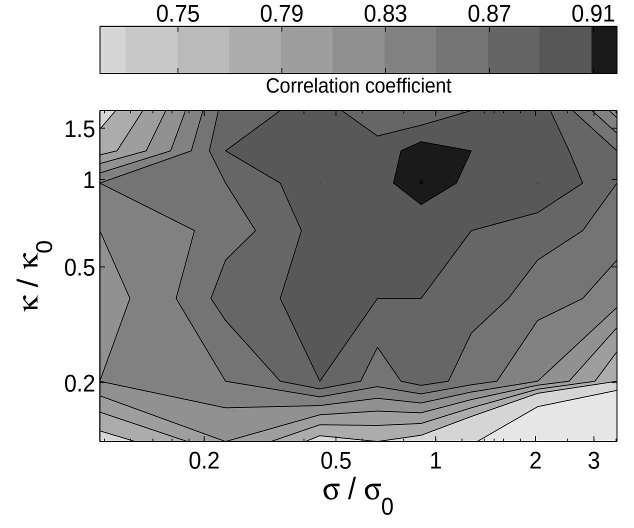

Figure 3: Phase space for mutual coupling. Correlation coefficient as a function of κ and σ, the self feedback and the coupling strength, respectively. κ_0 and σ_0 are equivalent to a 6.6% reduction in I_{th}^{sol}.

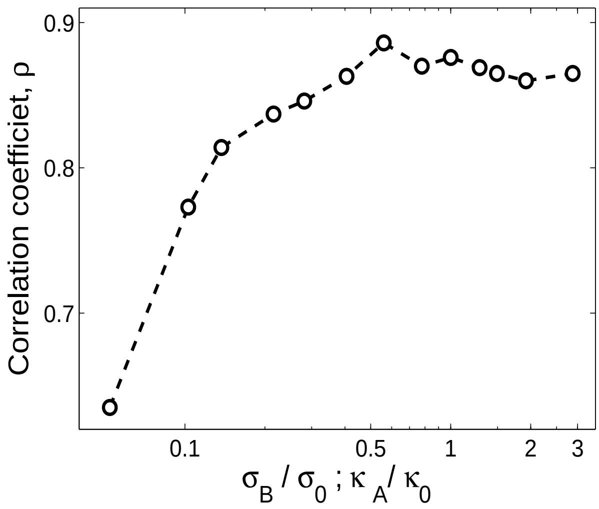

Figure 4: Correlation coefficient as a function of κ_A and σ_B, the self feedback of laser A and the coupling to laser B, respectively, for unidirectional coupling. $\kappa_A = \sigma_B$ at all time.

where I_A and I_B are the intensities of lasers A and B respectively (see note [22]). The best synchronization ($\rho \geq 0.91$) is achieved for feedback values of κ_0 and σ_0 (each equivalent to a 6.6% reduction in I_{th}^{sol}). Nevertheless, note that it is robust to changes in feedback strength and stable isochronal synchronization is maintained even for deviations of 50% from these values. This robustness is very encouraging considering real applications where channel intensity might suffer from fluctuations and disturbances. A theoretical phase-space, based on a numarical solution of the well-known Lang-Kobayashi equations is available in ref. [20] and confirms the existence of a broad area where the lasers exhibit stable synchronization. When considering the case of $\kappa = \sigma$, the stable synchronization is intuitively understood because each laser is subjected to a feedback made of a both laser intensities, hence their "driving force" is similar. However, we provide an experimental proof that this condition is not necessary and coupling intensity can be weaken which is favorable for the realization of secure information exchange [8, 9].

For the case of unidirectional coupling (see Fig.1), a phase space is presented in Fig.4. Here, the TL (laser A) is subjected only to a self feedback from its external cavity (κ_A) and the RL (laser B) is subjected only to coupling from laser A (σ_B), while the condition of a total feedback amount symmetry is kept ($\kappa_A = \sigma_B$). Similarly to mutual coupling, good synchronization is achieved in the vicinity of κ_0 and σ_0. This phase space is one dimensional because other wise the symmetry condition breaks. We considered an additional self feedback to the RL, also known as a closed loop scheme [7], however, all attempts resulted in deterioration of the synchronization. In the closed loop case, synchronization is very sensitive to the phase of the returning field, demanding a careful adjustment of the external cavity length to sub wavelength levels [7, 23]. We note that we have not seen this dependency for the mutually coupled lasers, however further investigation is required.

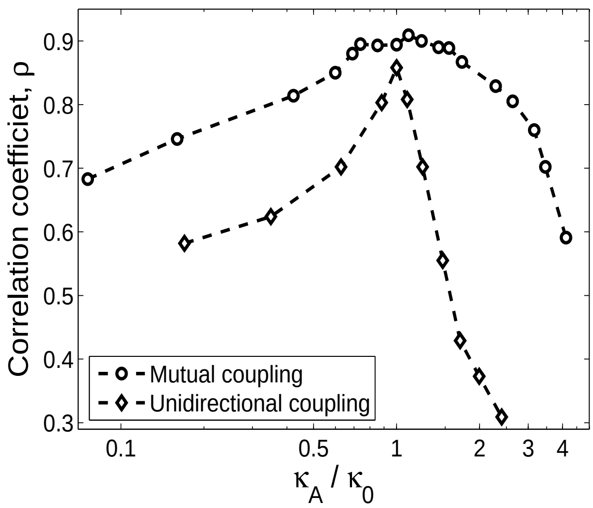

Figure 5: Correlation coefficient for different values of κ_A, laser A's self feedback rate. Coupling strength is σ_0. $\kappa_b = \kappa_0$ and $\kappa_b = 0$ in the mutual (open circuits) and unidirectional (open diamonds) coupling, respectively.

We turn to study sensitivity to the break in the total feedback amount symmetry for both unidirectional and mutual coupling configurations. Hereafter, unless stated otherwise, feedback values are kept at $\kappa = \kappa_0$ and $\sigma = \sigma_0$. In Fig.5 we show the influence of varying κ_A, on the quality of synchronization between the lasers. It

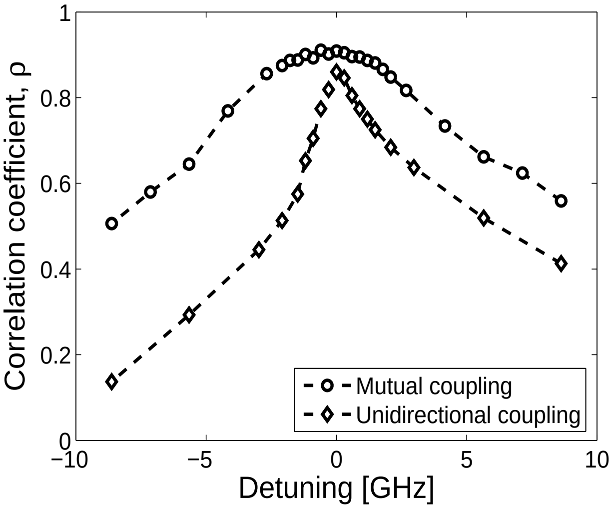

Figure 6: Correlation coefficient as a function of detuning of laser A's optical field's frequency in respect to laser B, for mutual (open circuits) and unidirectional (open diamonds) coupling. Feedback strengths are σ_0 and κ_0.

can be seen that for the unidirectional case (open diamonds), unless $\kappa_A = \kappa_0$, good synchronization cannot be attained. However, for mutual coupling (open circuits), small deviations of the self feedback strength of one laser in respect to that of the other, does not affect synchronization quality. In this case, good synchronization of $\rho = 0.9$ is available for the strength mismatch of $0.7\kappa_0 \leq \kappa_A \leq 1.5\kappa_0$. Thus mutual coupling exhibits high tolerance for asymmetry between the operating lasers, while unidirectional coupling is very sensitive to the mismatch and synchronization appears only if the condition of total feedback amount symmetry is fulfilled.

We also examined the sensitivity of the two schemes to detuning in the optical frequency of one laser in respect to the other. We changed laser A's temperature while keeping laser B's temperature constant. Varying our laser's temperature causes its gain curve to move at a rate of 0.17 nm/K. However, more important in this case is the shift of internal modes which we measured to be at a rate of 0.043 nm/K. The gain curve is wide enough to be neglected if temperature shift is of the order of a fraction of a degree K, whereas the mode matching is crucial for synchronization. In Fig.6, the detuning effect is presented. Note that unidirectional coupling scheme (open diamonds) is very sensitive for the change in one of the lasers' frequency. This result was perceived before in ref.[11, 12]. Mutual coupling, on the other hand, is robust and will easily suffer a detuning of ± 2GHz (± 0.003nm), equivalent to ± 0.07K. A semiconductor laser is known to execute a spectral line broadening of a few GHz when subjected to an external feedback [3, 4]. If one laser is detuned in respect to the other, this results in a smaller overlap between their spectra, and effectively reducing coupling strength σ. By comparing between the phase-space for mutual coupling in Fig.3 and the frequency detuning in Fig.6, one can tell that detuning of 2GHz between the lasers is equivalent to a reduction in σ by factor 2 where good synchronization is still observed. This also explains why unidirectional coupling is so sensitive to detuning: effectively, it means breaking the condition of total feedback amount symmetry, hence the similarity to Fig.5.

To conclude, we have investigated experimentally the phase-space of feedback and coupling strengths for two mutually coupled lasers each subjected to an external feedback. We have found that parameter choice for good synchronization is wide. A comparison between unidirectional and mutual coupling, for the identical synchronization case, reveled that while the first is extremely sensitive to deviations in the feedback strengths and frequency detuning, the latter shows robust synchronization even under non identical operation conditions and might be suitable for optical communication applications [9].

[1] G.H.M. van Tartwijk, A.M. Levine, and D. Lenstra, IEEE Journal of Selected Topics in Quantum Electronics, vol. 1, no. 2 (1995).
[2] T. Heil, I. Fischer, and W. Elsaber, Phys. Rev. A, vol. 58, no. 4 (1998).
[3] R.O. Miles, A. Dandridge, A.B. Tveten, H.F. Taylor, and T.G. Giallorenzi, Applied Physics Letters, vol. 37, no. 11 (1980).
[4] D. Lenstra, B.H. Verbeek, and A.J. Den Boef, IEEE Journal of Quantum Electronics, vol. 21, no. 6 (1985).
[5] J.M. Buldu, J. Garcia-Ojalvo, and M.C. Torrent, IEEE Journal of Quantum Electronics, vol. 40, no. 6 (2004).
[6] A. Argyris, et al. Nature 438, 343-346 (2005).
[7] C.R. Mirasso, R. Vicente, P. Colet, J. Mulet, and T. Perez, C. R. Physique 5, 613-622 (2004).
[8] E. Klein, R. Mislovaty, I. Kanter and W. Kinzel, Phys. Rev. E, vol. 72, 016214 (2005).
[9] E. Klein, N. Gross, E. Kopelowitz, M. Rosenbluh, W. Kinzel, L. Khaykovich and I. Kanter. (Arxiv)
[10] A. Murakami, and J. Ohtsubo, Phys. Rev. A, vol. 65, 033826 (2002).
[11] R. Vicente, T. Perez, and C.R. Mirasso, IEEE Journal of Quantum Electronics, vol. 38, no. 9 (2002).
[12] Y. Liu, P. Davis, Y. Takiguchi, t. Aida, S. Saito, and J.M. Liu, IEEE Journal of Quantum Electronics, vol. 39, no. 2 (2003).
[13] Y. Liu, H.F. Chen, J.M. Liu, P. Davis, and T. Aida, IEEE Transactions on Circuits and Systems-I, vol. 48, no. 12 (2001).
[14] Y. Liu, Y. Takiguchi, P. Davis, T. Aida, S. Saito, and J.M. Liu, Applied Physics Letters, vol. 80, no. 23 (2002).
[15] T. Heil, J. Mulet, I. Fischer, C.R. Mirasso, M. Peil, P. Colet, and W. Elsäßer, IEEE Journal of Quantum Electronics, vol. 38, no. 9 (2002).

[16] D. Kanakidis, A. Agryris, A. Bogris, and D. Syvridis, Journal of Lightwave Technology, vol. 24, no. 1 (2006).
[17] T. Heil ,I. Fisher, W. Elsässer, J. Mulet and C.R. Mirasso, Phys. Rev. Lett. vol. 86, 795 (2001);
[18] J. Mulet, C. Mirasso, T. Heil, and I. Fischer, J. Opt. B: Quantum Semiclass. Opt. 6 (2004) 97-105.
[19] H. Erzgräber, D. Lenstra, B. Krauskopf, E. Wille, M. Peil, I. Fischer, and W. Elsäßer, Opt. Comm. 255 (2005) 286-296.
[20] E. Klein, N. Gross, M. Rosenbluh, W. Kinzel, L. Khaykovich and I. Kanter, cond-mat/0511648 (To appear in Phys. Rev. E.)
[21] N.A. Olsson, and N.K. Dutta, Applied Physics Letters, vol. 44, no. 9 (1984).
[22] We work in the well studied LFF (Law Frequency Fluctuations) regime[1–4], characterized by intensity breakdowns. These breakdowns occur on a time scale of hundreds of ns while the chaotic features we are interested in are an order of magnitude smaller in intensity and fluctuate on time scales of ns (our resolution limit). In order to get the most accurate result for overlap, we divide the intensity traces into 10ns segments (containing 10 points of sample) and the correlation coefficient is calculated between matching segments and then averaged over all (including the intensity breakdowns themselves).
[23] M. Peil, T. Heil, I. Fischer, and W. Elsäßer, Phys. Rev. Lett. 88, 174101 (2002).